Chapter 2
Reasoning and Proof

Section 4
Reasoning with Properties from Algebra

GOAL 1: Using Properties from Algebra

Many properties from algebra concern the equality of real numbers. Several of these are summarized in the following list.

ALGEBRAIC PROPERTIES OF EQUALITY

Let a, b, and c be real numbers.

ADDITION PROPERTY	If $a = b$, then $a + c = b + c$.
SUBTRACTION PROPERTY	If $a = b$, then $a - c = b - c$.
MULTIPLICATION PROPERTY	If $a = b$, then $ac = bc$.
DIVISION PROPERTY	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
REFLEXIVE PROPERTY	For any real number a, $a = a$.
SYMMETRIC PROPERTY	If $a = b$, then $b = a$.
TRANSITIVE PROPERTY	If $a = b$ and $b = c$, then $a = c$.
SUBSTITUTION PROPERTY	If $a = b$, then a can be substituted for b in any equation or expression.

Properties of equality along with other properties from algebra, such as the distributive property,

$$a(b + c) = ab + ac$$

can be used to solve equations. For instance, you can use the subtraction property of equality to solve the equation $x + 3 = 7$. By subtracting 3 from each side of the equation, you obtain $x = 4$.

Example 1: Writing Reasons

Solve $5x - 18 = 3x + 2$ and write a reason for each step.

$$\cancel{-18} + \cancel{18} \qquad +18 \qquad \text{Addition}$$

$$5x = \cancel{3x} + 20$$

$$-3x \quad \cancel{-3x} \qquad \text{Subtraction}$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{20}{2} \qquad \text{Division}$$

$$x = 10$$

Example 2: Writing Reasons

Solve $55z - 3(9z + 12) = -64$ and write a reason for each step.

$55z - 27z - 36 = -64$	Distribute
$28z - 36 = -64$	C.L.T.
$\quad +36 \quad +36$	Addition
$28z = -28$	
$28 \qquad 28$	Division
$z = -1$	

Example 3: Using Properties in Real Life

Fitness: Before exercising, you should find your target heart rate. This is the rate at which you achieve an effective workout while not placing too much strain on your heart. Your target heart rate *r* (in beats per minute) can be determined from you age *a* (in years) using the equation $a = 220 - \frac{10}{7}r$.

a. Solve the formula for r and write a reason for each step.

b. Use the result to find the target heart rate for a 16 year old.

c. Find the target heart rate for the following ages: 20, 30, 40, 50, and 60. What happens to the target heart rate as a person gets older?

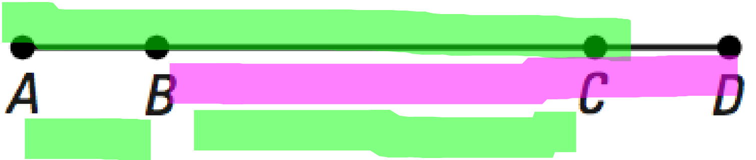
GOAL 2: Using Properties of Length and Measure

The algebraic properties of equality can be used in geometry.

CONCEPT SUMMARY — **PROPERTIES OF EQUALITY**

	SEGMENT LENGTH	ANGLE MEASURE
REFLEXIVE	For any segment AB, $AB = AB$.	For any angle A, $m\angle A = m\angle A$.
SYMMETRIC	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
TRANSITIVE	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Example 4: Using Properties of Length



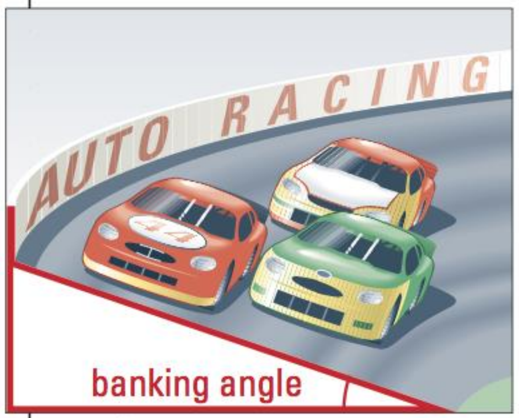
In the diagram, AB = CD. The argument below shows that AC = BD.

AB = CD	Given
AB + BC = BC + CD	Addition P.O.E.
AC = AB + BC	Segment Addition Postulate
BD = BC + CD	Segment Addition Postulate
AC = BD	Substitution P.O.E.

Example 5: Using Properties of Measure

Auto Racing: The Talladega Superspeedway racetrack in Alabama has four banked turns, which are described in the diagram. Use the given information about the maximum banking angle of the four turns to find m<4.

FOCUS ON APPLICATIONS



AUTO RACING Banked turns help the cars travel around the track at high speeds. The angles provide an inward force that helps keep the cars from flying off the track.

$$m\angle 1 + m\angle 2 = 66^\circ$$

$$m\angle 1 + m\angle 2 + m\angle 3 = 99^\circ$$

$$m\angle 3 = m\angle 1$$

$$m\angle 1 = m\angle 4$$

$m\angle 1 + m\angle 2 = 66°$	Given
$m\angle 1 + m\angle 2 + m\angle 3 = 99°$	Given
$66° + m\angle 3 = 99°$	Substitution
$m\angle 3 = 33°$	Subtraction
$m\angle 3 = \cancel{m\angle 1}, \cancel{m\angle 1} = m\angle 4$	Given
$m\angle 3 = m\angle 4$	Transitive
$m\angle 4 = 33°$	Substitution